Graphs are used to represent relationships between two variables. They help us understand the relationships and often help us generalize beyond the data supplied. In functions, a value of $y$ is assigned to each value of $x$. The primary relationship in a function is that the $y$-value depends on the $x$-value. For this reason, the $x$-axis is called the independent axis and the $y$-axis is called the dependent axis. The relationship can also be seen in terms of slope, as the rate of change between the dependent $y$-axis and the independent $x$-axis; $\frac{\text{dependent } \Delta y}{\text{independent } \Delta x}$.

**Drawing the Graph of a Situation** In dealing with real situations, it becomes necessary to think of the $x$-axis as some independent variable, such as time, and to think of the $y$-axis as a dependent variable, such as height or distance. Whenever it is clear that one variable depends on the other, the dependent variable should be the $y$ and the independent variable should be the $x$. If a graph is comparing air pressure at different altitudes, then the pressure ($y$) depends on the altitude ($x$). But if a graph is charting a mountain climber's progress, then the altitude ($y$) depends on the time spent climbing ($x$).

*8th Grade Post-March Topic*
Deluxe Limousine Service charges $2 for the initial pickup of a passenger and then $1 per mile. Make a graph of this situation.

SOLUTION

Note that the fee for the ride depends upon the number of miles traveled. Therefore, the fee will be represented by the variable $y$ and distance will be represented by the variable $x$. The change per mile is the fraction that represents the change in price per change in miles traveled, or $\frac{\Delta y}{\Delta x}$, the slope, which equals $\frac{\$1}{1 \text{ mile}}$ or 1. The equation will then be $y = 2 + \$1x$.

We can make a table of miles and fees for Deluxe Limousine Service. The label for the $x$-axis is miles traveled and the label for the $y$-axis is taxi fee in dollars.

<table>
<thead>
<tr>
<th>Miles Traveled</th>
<th>Taxi Fee in Dollars</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

The graph starts where the number of miles traveled is 0. The fee at zero miles is $2. Thus the point (0, 2) is on the graph. A 1-mile trip costs $3 and a 2-mile trip costs $4. When we plot these points, we see a linear pattern. We can extend the graph in the first quadrant and we can read the fees for longer trips.

Answer:

Note: The graph is in the first quadrant only. Why does the graph stop at the $y$-axis?

Reading the Graph of a Situation On any math exam, you may be given the graph of a situation and then asked various questions about the situation. When answering these questions, remember:

- Questions that involve rates (time per room painted, daily charge) or speeds (miles per hour, births per year) will usually require information about the slope.
- Questions that involve starting times, opening deposits, beginning locations, or similar initial conditions are usually asking questions about the $y$-intercept.
- Questions that start “When will...” or “How many...” usually provide a value for one variable and expect you to find the value for the other variable from the corresponding point on the graph.
MODEL PROBLEM

This graph represents the billing structure for Friendly Taxi. What is the initial pickup fee and charge per mile for this service?

SOLUTION

The initial pickup fee is the charge before any driving is done, in other words when \( x \) is 0. The point on the graph with an \( x \) value of zero is the origin, (0, 0). Therefore, the pickup fee is $0.

The charge per mile is the change in price per change in miles, or \( \frac{\Delta y}{\Delta x} \), the slope. Two points on the line are (0, 0) and (2, 3), so \( \frac{\Delta y}{\Delta x} = \frac{3 - 0}{2 - 0} = \frac{3}{2} \approx 1.5 \) dollars per mile.

Answer: There is no pickup fee, and the charge per mile is $1.50.

Situations Involving Graphs of Systems
When two equations are graphed on the same coordinate plane, many types of questions involve comparing the graphs.

- Questions that ask “When is situation A better/lower than situation B?” want you to find an inequality involving \( x \) that describes what part of the graph satisfies the situation. For example, the answer might be “When the time is longer than 5 days.”
- Questions that ask “When are the values the same?” want you to find the coordinates of the point of intersection.
- Questions that ask about differences at specific values of \( x \) require you to subtract the \( y \)-value of one line from the other.

MODEL PROBLEM

Deluxe Limousine Service charges $2 for the initial pickup of a passenger and then $1 per mile. By comparison, Friendly Taxi does not have an initial pickup charge but charges $1.50 per mile. Graph the fee schedule for Deluxe Limousine on the same coordinate plane as the graph for Friendly Taxi. Then answer the following questions:

a. For what length trips is it cheaper to use Friendly Taxi?

b. At what mileage is the cost the same for both services? What is the cost?

c. What is the difference in cost at the 6-mile mark?
SOLUTION

The graphs of the costs of each service were discussed earlier in this section. By graphing them on the same coordinate plane, you can answer the three questions.

Answers:

a The graph for Friendly Taxi is below the graph for Deluxe Limousine up to the 4-mile mark. So for trips under 4 miles long, it is cheaper to use Friendly Taxi.

b The lines intersect at (4, 6). So at 4 miles, the cost for both services is $6.

c At 6 miles, the graph shows the cost for Deluxe Limousine is $8 and the cost for Friendly Taxi is $9. The difference is $9 - $8 = $1.

Practice

1 The graph represents the number of books read by Claire and Terry over a 4-week period. Who reads faster and by how much?

(1) Claire reads 8 more books per week than Terry.

(2) Claire reads 2 more books per week than Terry.

(3) Claire reads 1 more book per week than Terry.

(4) Claire reads twice as many books per week as Terry.
2 Alyssa bicycled for 2 hours at 4 miles per hour. She stopped for one hour to visit a friend. She bicycled for another hour at 5 miles per hour. Which graph best represents Alyssa's trip if the horizontal axis is time and the vertical axis is distance?

3 Both Janine and Fran used exercise equipment at their gym. Fran walked on a treadmill and Janine rode a stationary bicycle. Their times and calories burned are shown in the graph below. In calories burned per hour, how much faster was Janine than Fran?

4 The figure below shows the cost of membership for two recreational clubs. For each club, the cost includes an initial fee to join the club plus a monthly charge.

   a What is the initial fee for Club A? For Club B?
   b For which month will the total expenses be the same for both clubs? What is that total cost?
   c What is the monthly charge for club A? For Club B?
5 Two different health clubs have the following rates. Sammy's Spa charges a flat fee of $350 a year for the use of the club, machines, the pool, and the classes. Shape Up! charges $150 a year for the use of the club, machines, and pool, plus $20 per exercise class.

a Write an equation to represent each health club's yearly fees.
b Graph each equation with appropriate labels for the axes. **Skip.**
c Under what circumstances is it more economical to join Shape Up? **Skip.**

6 The graph below shows the relationship between the depreciated value \( y \) of a car (in thousands of dollars) and the passage of time \( t \).

\[
\begin{array}{c|c|c|c|c|c|c|c|c|c}
\hline
\text{Time (in years)} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
\hline
\text{Value (in thousands $)} & 22 & 16 & 10 & 4 & 0 & 4 & 8 & 12 & 16 \\
\hline
\end{array}
\]

a Write a linear equation to represent the depreciated value of the car.
b Use that equation to find the value of the car after 4.5 years.

7 The graph below shows the relationship between the cost, \( C \), (in thousands of dollars) of manufacturing DVD players and the number of DVD players produced, \( N \).

\[
\begin{array}{c|c|c|c|c|c|c|c|c|c}
\hline
\text{Number of DVD Players} & 0 & 50 & 100 & 150 & 200 & 250 & 300 & 350 \\
\hline
\text{Cost (in thousands $)} & 6 & 12 & 18 & 24 & 30 & 36 & 42 & 48 \\
\hline
\end{array}
\]

a Write a linear equation to represent the cost of manufacturing the DVD players.
b Use that equation to find the cost of manufacturing 300 DVD players.

8 George goes walking at noon at a rate of 3 miles an hour. At 2 P.M., Pete follows George on a bicycle at the rate of 5 miles an hour. The graph below shows both the time (in hours) and distance (in miles) of both George and Pete.

\[
\begin{array}{c|c|c|c|c|c|c|c|c|c}
\hline
\text{Hours} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\hline
\text{Miles} & 0 & 3 & 6 & 9 & 12 & 15 & 18 & 21 \\
\hline
\end{array}
\]

a Write a linear equation to represent the distance traveled in terms of rate and time for each person.
b After how many hours does Pete finally catch up to George?
c After how many more hours will George be 8 miles behind Pete? **Skip.**